Economic Analysis of Havven's White Paper

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Executive Summary

In their White Paper, the Havven foundation (Havven) describe a new cryptocurrency designed to be stable i.e. to have a constant exchange rate to a fiat currency such as USD.

The design relies on a dual-token system: a reserve token (named havven) which serves as collateral for an exchange token (named nomin), which can be used for transactions.

In order to stabilize the price of the nomins, the owners of havvens are incentivized to change the supply of nomins in reaction to demand fluctuations.

This report analyzes the incentives of all involved players from a game theoretic perspective and investigates the effect of demand shocks through numeric simulations.

Our analysis shows that under certain assumptions havven holders behave as intended by the system and the price of nomins stabilizes around the target value.

All underlying assumptions are reasonable and reflect the best of our knowledge at the time of writing this report. However, the results may change fundamentally if critical assumptions underlying the analysis are violated. In particular,

- demand for nomins was modelled as a mathematical function with characteristics that allow certain equilibria to arise.
- certain parameter values are crucial for the stability of the system.
- Havven, who design and implement the mechanism, do not require any incentives to fulfill their role and act in the best interest of all players.

1 Introduction

Havven aims to launch a new cryptocurrency that is designed to reduce the volatility associated with most other cryptocurrencies, such as Bitcoin. It creates a token which exhibits a stable exchange rate with respect to an external asset (e.g. USD). As a result, the new token could operate as both a means of exchange and a unit of account.

Havven proposes a dual-token solution which is composed of a stabilized exchange token (named nomin) and a reserve token (named havven) which provides collateral for the issuance of the nomin. Havven holders are incentivized to change the supply of nomins, so that no central authority is needed to maintain a stable exchange rate. Havven produced a White Paper which describes the main characteristics of the solution and the underlying incentive mechanism.

The center for cryptoeconomics (cryptecon) supported Havven in designing the incentive scheme which governs the supply of nomins. The aim was to advance the White Paper to a point where Havven and cryptecon felt confident that the mechanism will produce a stable currency.

Based on the descriptions in the White Paper, cryptecon conducted a detailed analysis of the Havven economy and the underlying incentive scheme in particular. Cryptecon developed a game theoretic model in order to validate the incentive scheme underlying supply and demand for havvens and nomins. The model studies the partial equilibria that arise under different scenarios. Finally, cryptecon also conducted a numerical simulation to test the stability of the identified equilibria.

The results of cryptecon's economic analysis are subsequently presented. In Section 2 we describe the proposed stabilization mechanism and formalize the economic relationships in a game theoretic model. In Section 3 we proceed with two different but complementary types of equilibrium analysis: an analytical approach in Subsection 3.1 and a calibrated approach in 3.2. In Section 4 we discuss the assumptions underlying the model, the involved players' incentives and the main results. We also highlight certain issues and risks that may deserve closer attention from Havven. Section 5 presents the results from the numerical simulations. Finally, we conclude and summarize all recommendations in Section 6.

2 Model

We start by describing the general framework of the Havven economy. The formal model is based on the Section "System Description" of Havven's White Paper.

The players: There are three main players and two tokens. The first player is a principal (hereafter "the foundation") who issues and sells a fixed number H of the currency's reserve token havven at price $P_h \in \Re^+ \cup \{0\}$ in $[USD/H]$.

Buyers of havvens (hereafter "havven holders") constitute the second player in the model. They have the right to issue (and sell) nomins using their havvens as collateral. They are rewarded for issuing with transaction fees (which will be explained in detail below). They also have the right to buy back nomins they have issued in case they want to sell their escrowed havvens. Nomins are the exchange token and can be used for trade purposes by "nomin users" (defined below). We denote the aggregate number of nomins by N.

Nomin users constitute the third player in the model. They can buy and sell nomins at a price $P_n \in \Re^+ \cup \{0\}$ (in $[USD/N]$) in the market and use them for transactions. A transaction fee is paid by nomin users, which is sufficiently small so that it provides little to no friction on the demand for nomins. The transaction fees are distributed to those havven holders that have issued nomins, as a reward for their role in maintaining a stable nomin price. Thus, the larger the number of transactions with nomins, the larger the reward for havven holders.

The foundation is assumed to act like a benevolent dictator (i.e. a principal whose goal it is to maximise the overall welfare of all market participants) and designs a set of incentives (the mechanism) to stabilize the price of nomins at $1\,USD$ per nomin.[1] In particular, this set of incentives induces havven holders to (i) provide the collateral and (ii) participate in the stabilization of the nomin exchange rate.

Havven holders actions: The model is designed as an infinite game with discrete time periods. At any moment in time t, havven holder i chooses to issue a total quantity of nomins $N_{i,t}$ which go into circulation in the market. The net amount of nomins he issues in period t is defined as the difference $N_{i,t} - N_{i,t-1}$. A havven holder decides between issuing nomins (i.e., $N_{i,t} > N_{i,t-1}$), buying back nomins (i.e., $N_{i,t} < N_{i,t-1}$), or remaining idle (i.e., $N_{i,t} = N_{i,t-1}$). When a havven holder issues nomins, he does not decide whether to sell them or not. Upon issuance, nomins are sold automatically by the foundation at a minimum price $P_{n,t}$ of 1. When a havven holder buys nomins, he does it at a maximum price $P_{n,t}$ of 1. A havven holder cannot sell one of his havvens if there are nomins in circulation which are collateralized by this havven.

Imposing the maximum and minimum price for buying and selling nomins, respectively, implies that there is no room for nomin users to speculate inducing changes in P_n. However, a havven holder may achieve gains from "seigniorage", receiving an expected profit of $\int_{N_{i,t-1}}^{N_{i,t}} N_{i,t} P_{n,t}(N_{i,t}) dN_{i,t}$.

Collateralization ratios: Each havven holder has the right to issue a number of nomins. However, the value of his issued nomins cannot be larger than the value of the havvens he holds, i.e., $P_{n,t}N_{i,t} \leq P_{h,t}H_{i,t}$, for all i and t, where $P_{h,t}$ is the havven price at period t. These rules hold for every havven holder i, and thus $P_{n,t}N_t \leq P_{h,t}H$, where $N_t = \sum_i N_{i,t}$

[1] Although in the real world nomins and havvens are going to be traded in ETH, since the reference for the price of a nomin is denominated in USD, we will proceed with the analysis taking $P_n = 1\,USD/N$ as the intended constant nomin price.

and $H = \sum_i H_{i,t}$.[2] The goal of this rule is to have enough collateral to buy back nomins when needed. Thus, the foundation continuously tracks the collateralization ratio of every i:

$$C_{i,t} \equiv \frac{P_{n,t} N_{i,t}}{P_{h,t} H_{i,t}}. \tag{1}$$

Besides the aggregate collateral ratio $C_t = (P_{n,t} N_t)/(P_{h,t} H)$, the mechanism imposes two additional thresholds on the collateralization ratio: (i) the optimal collateralization ratio $C_{opt,t}$, and (ii) the maximum collateralization ratio $C_{max,t}$. The former is the target ratio which maintains a stable nomin price and maximizes a havven holder's profits from fees. The latter reduces the risk of a collapse due to an undercollateralization (i.e., when the value of nomins becomes larger than the value of value of the collateral).

The relation between the three ratios is the following:

$$C_{opt,t} \equiv f(P_{n,t}) C_t, \tag{2}$$
$$C_{max,t} \equiv a C_{opt,t}, \tag{3}$$

where $a \geq 1$. The function $f(P_{n,t})$ has the following properties: $f(P_{n,t}) \geq 0$, $f'(P_{n,t}) \geq 0$, $f'(P_{n,t}) = 0$ at $P_{n,t} = 1$, $f''(P_{n,t}) < 0$ for $P_{n,t} < 1$, and $f''(P_{n,t}) > 0$ for $P_{n,t} > 1$. Thus, when the nomin price falls below $1\,USD$ (e.g. due to a reduction in the demand for nomins), the C_{opt} required decreases. Although C_{t+1} also decreases with respect to C_t, it does so at a lower rate than $C_{opt,t+1}$. Thus, havven holders need to buy and burn nomins to get $C_{t+1} = C_{opt,t+1}$. On the other hand, when the price of nomin climbs above the price of $1\,USD$, the new $C_{opt,t+1}$ increases at a larger rate than C_{t+1}, giving the havven holder incentives to issue additional nomins.

In particular, the function $f(P_{n,t})$ takes the form

$$f(P_{n,t}) \equiv max\{\sigma (P_{n,t} - 1)^{\phi} + 1, 0\}. \tag{4}$$

The fee functions: Havven holder i's incentive to issue nomins arises from the transaction fees he receives. Every nomin user has to pay a small percentage α_c for each transaction. [3] In particular, the aggregate transaction fees collected by the mechanism at any period t depends on the number of nomins issued and on the velocity of their circulation,

$$F_t \equiv \alpha_c v_t N_t. \qquad [N] \tag{5}$$

The aggregated fees are distributed among the havven holders who have issued nomins. The fees are the reward for stabilizing the system. They are distributed to havven holders

[2] The total number of havvens is fixed accross periods. Thus, $H = H_t = \sum_i H_{i,t}$

[3] This fee may become smaller with an increased adoption of nomins.

depending on their collateralization ratio, the optimal and maximal collateralizations thresholds, and the number of havvens they hold. In particular, the fees per havven collected by i in period t are

$$\alpha_{r,i,t} = \alpha_{base,t}\mathit{\Gamma}_{i,t}, \qquad [N/H] \tag{6}$$

where

$$\mathit{\Gamma}_{i,t} \equiv \begin{cases} \frac{C_{i,t}}{C_{opt,t}} & \text{if } C_{i,t} \leq C_{opt,t}, \\ \\ \frac{C_{max,t}-C_{i,t}}{C_{max,t}-C_{opt,t}} & \text{if } C_{opt,t} \leq C_{i,t} \leq C_{max,t}, \\ \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

In the case of $C_{max,t} < C_{i,t}$, havven holder i does not collect any fees. The aggregate amount of fees collected from nomin users has to be equal to the total amount of fees rewarded to havven holders, i.e.,

$$F_t = \sum_i \alpha_{r,i,t} H_{i,t}. \qquad [N] \tag{8}$$

This last equality yields the floating value of $\alpha_{base,t}$,

$$\alpha_{base,t} = \frac{\alpha_c \upsilon_t N_t}{\sum_i H_i \mathit{\Gamma}_{i,t}}. \qquad [N/H] \tag{9}$$

Escrowed Havvens: As mentioned above, a havven holder must not sell havvens which act as collateral for nomins in circulation. The number of escrowed havvens for each havven holder is defined as

$$\check{H}_{i,t} = \frac{P_{n,t} N_{i,t}}{P_{h,t} C_{max,t}}. \tag{10}$$

Havven holder i may sell his remaining havvens $H_{i,t} - \check{H}_{i,t}$ without restriction. However, he has to take into account that this action will change his collateralization ratio and, as a consequence, his reward from fees.

Main assumptions: It is important to restate certain assumptions underlying (implicitly or explicitly) the mechanism proposed in the White Paper.

Assumption 1. *The price of each havven is given by*

$$P_{h,t} \equiv \frac{1}{H} \sum_{\tau=1}^{\infty} \frac{F_t P_{n,t}}{(1+R)^{\tau}} = \frac{F_t P_{n,t}}{HR}. \qquad [USD/H] \tag{11}$$

Important: In the White Paper, Formula 11 is written as

$$P_{h,t} \equiv \frac{F_t}{HR}.$$

Hence, the price of havven is defined in terms of nomins when it should be defined in terms of USD. We recommend to fix this.

This assumption comes with two implications. First, it reduces the mechanism's exposure to speculative attacks through $P_{h,t}$ since the set of incentives does not depend on the market price of havvens but on the price given by this assumption. Second, it allows for an explicit relationship between the demand for nomins, return from fees, and the price of havvens. This relationship will be useful for the subsequent analysis.

Assumption 2. *The foundation does not play strategically.*

The set of incentives for the foundation is exogenous to the model i.e. within the scope of the subsequent analysis, the foundation does not look for an optimal mechanism, but takes a passive role in controlling the stability of the system.

Assumption 3. *There exists a positive demand for nomins.*

This assumption is crucial. Without it we cannot proceed with the analysis. The assumption will be discussed in Section 4 in more detail.

The havven holder's problem: As a consequence of assumptions 2 and 3, only havven holders behave strategically.

A havven holder i planning to sell havvens after T periods makes profits from three different sources: (i) fees rewarded from his collateralization ratio up to period T, (ii) future sales of his havvens, and (iii) seigniorage. Since in equilibrium the market price of a havven must be equal to the expected discounted future fees rewarded, the profit from selling a havven in period $t+T$ is equal to $P_{h,t+T}$. Therefore, we can consider that havven holder i's expected profits only depends on fees and seigniorage.

Hence, at any period t, any havven holder i chooses $N_{i,t}$ to maximize his expected payoffs. He does it while taking into account the mechanism proposed and other havven holders' behavior, i.e., $N_{-i,t}$ (where $-i$ indicates all havven holders but i). At any period t, havven holder i observes the price of the previous period $P_{n,t-1}$, which together with $N_{i,t-1}$, $P_{h,t-1}$ and H_i, gives $C_{i,t-1}$ and $C_{opt,t-1}$. Thus, he chooses $N_{i,t}$ to solve

$$\max_{N_{i,t}} \sum_{\tau=0}^{\infty} \frac{1}{(1+R)^{\tau+1}} \left[\int_{N_{i,t+\tau-1}}^{N_{i,t+\tau}} N_{i,t+\tau} \tilde{P}_{n,t+\tau}(N_{t+\tau}) dN_{i,t+\tau} + \alpha_{r,i,t+\tau} H_{i,t+\tau} \right], \tag{HHP}$$

subject to,

$eq.(1) - eq.(11)$,

$$\tilde{P}_{n,t+\tau}(N_{t+\tau}) \equiv \begin{cases} P_{n,t+\tau}(N_{t+\tau}) & \text{if } N_{i,t+\tau} > N_{i,t+\tau-1} \text{ and } P_{n,t+\tau}(N_{t+\tau}) \geq 1, \\ P_{n,t+\tau}(N_{t+\tau}) & \text{if } N_{i,t+\tau} < N_{i,t+\tau-1} \text{ and } P_{n,t+\tau}(N_{t+\tau}) \leq 1, \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

$$\begin{aligned} N_{i,t+1} \equiv \arg\max_{N_{i,t+1}} \sum_{\tau=1}^{\infty} \frac{1}{(1+R)^{\tau}} \int_{N_{i,t+\tau}}^{N_{i,t+\tau+1}} & N_{i,t+\tau+1} \tilde{P}_{n,t+\tau+1}(N_{t+\tau+1}) dN_{i,t+\tau+1} + \\ & + \sum_{\tau=1}^{\infty} \frac{1}{(1+R)^{\tau+1}} \alpha_{r,i,t+\tau+1} H_{i,t+\tau+1}, \end{aligned} \tag{13}$$

subject to $eq.(1)_{t+1} - eq.(12)_{t+1}$,

where $P_{n,t+\tau}(N_{t+\tau})$ is the inverse demand function for nomins at period $t+\tau$. Note that we are assuming that the havven holder receives the rewards at the end of each period.

At period t, the havven holder chooses $N_{i,t}$ to maximize his expected discounted profits (objective function (HHP)) knowing that in the next period he will choose the optimal $N_{i,t+1}$ (eq.(13)). Eq. (12) indicates the restriction in the maximum and minimum price at which a havven holder has to buy and sell nomins, respectively.

3 Analysis

In this section we proceed with the analysis of the previous setup. In particular, we concentrate our analysis on the havven holder's problem. In reality, the demand for nomins is unknown (at least in the beginning when no information for a reasonable estimate is available yet). However, we need to assume a particular demand function to proceed with the analysis. We use the quantity theory of money, in particular Fisher's equation of exchange, to define the demand for nomins function. This equation states that the demand for money (L) is:

$$L = \frac{GDP}{v}.$$

In equilibrium, the demand for money equals the supply of money, and therefore the following condition must hold:

$$L = N.$$

We consider that a fraction (ε) of the total amount of world e-commerce (which we denote as our GDP) will use nomins as the transaction currency. Since the GDP is measured in USD, the Fisher equation becomes:

$$vN = \frac{\varepsilon GDP}{P_n}.$$

This equation can be rearranged to resemble the inverse demand function for nomins:

$$P_n = \frac{\varepsilon GDP}{vN}. \qquad [USD/N] \tag{14}$$

The analysis is conducted using two different approaches. First, we solve the havven holder's problem (HHP) when each havven holder knows the demand function for nomins. This is the analytical approach. Second, we assume that havven holders do not know the demand function for nomins, but can observe the price of a nomin at any point in time.[4] For this case, we use a calibrated approach. By that we do not mean that we try to define correct values for the various parameters. The name rather hints at the havven holders' behaviour. In this approach, havven holders calibrate their number of nomins in a sequence of steps to maximize their fees. Each havven holder observes the collateralization levels and, as consequence, adjusts his number of nomins to increase his fees.[5] This yields a new nomin price and collateralization levels. Thus, havven holders may have incentives to adjust their amount of nomins again and so on.

Before the detailed analysis, we present the main results:

Main Result: Under certain assumptions and parameters, havven holders will follow the proposed mechanism and the price of nomins will stabilize at around $P_n = \$1$.

3.1 Analytical approach:

We have formulated the havven holder's problem as an infinite game since, if he decides to sell his havvens, the price he will receive is given by (11). Thus, the game can be solved recursively, i.e. we look for the solution of period t assuming that the havven holder behaves optimally in the following periods.

We consider the current period (which we normalize as $t = 0$) in which havven holder i observes that the price of nomins in the previous period $P_{n,-1} \neq 1$ ($C_{i,-1} \neq C_{opt,-1}$). We want to see if he chooses the number $N_{i,0}$ such that $P_{n,0} = 1$ and restores $C_{i,0} = C_{opt,0}$ when all other havven holders are choosing $N_{-i,0}$.[6] Note that, when i modifies $N_{i,0}$, he is

[4]This will also be the case in reality, which makes the assumption reasonable.

[5]Since his collateralization level C_i and collateralization target C_{opt} differ only when $P_n \neq 1$, it is equivalent to say that the havven holder observes P_n.

[6]For simplicity, we consider that there is only one additional havven holder. This is equivalent to a situation with many havven holders with homogeneous behaviour.

not only changing $P_{n,0}$ but also $C_{i,0}$ and $C_{opt,0}$ (through $f(P_{n,0})$ and C_0). Hence, he has to take into account all these effects which affect his rewarded fees.

Moreover, havven holder i would like to increase his rewarded fees, but he would also like to choose a $N_{i,0}$ to get profits from seigniorage. For instance, when $P_{n,0}$ is low, he may try to purchase nomins to induce undersupply of nomins and $P_{n,0} > 1$. Later, he could sell the excess number of nomins at his hand at that price.

In addition to the assumptions from White Paper, which are outlined above, we require the following additional assumptions for the analytical approach:

Assumption 4. *Havven holders sell their havvens in the third period at price $P_{h,2}$. The new holders instantaneously sell nomins at price $P_{n,2} = 1$.*

We make this assumption for two reasons. First, it allows us to simplify the problem to a two-period case. Since in the last period the trade of havvens is done at the price of nomins of 1, there is no strategic decision during that period. Additionally, two periods allows us to understand whether the havven holder has incentives to manipulate the number of nomins to get gains from seigniorage.

Assumption 5. *The function $f(.)$ has value zero when $P_n = 0$, i.e., $f(0) = 0$.*

The assumption helps to avoid a case in which $f(\varepsilon) >> 0$ when ε is close to zero, which would imply $C_{opt} > C$ (inducing an increment in N) when a reduction in N is needed to increase P_n. This last assumption requires $\sigma \geq 1$.

Assumption 6. *Havven holder i observes the total number of nomins N_{-i} issued by all other havven holders.*

Assumption 7. *Each havven holder knows the inverse demand function for nomins, which has the general shape of equation (14).*

Therefore, problem (HHP) becomes:

$$
\begin{aligned}
\max_{N_{i,0}} \frac{1}{(1+R)}[\int_{N_{i,-1}}^{N_{i,0}} N_{i,0}\tilde{P}_{n,0}(N_0)dN_{i,0} + \alpha_{r,i,0}H_{i,0}] + \\
+ \frac{1}{(1+R)^2}[\int_{N_{i,0}}^{N_{i,1}} N_{i,1}\tilde{P}_{n,1}(N_1)dN_{i,1} + \alpha_{r,i,1}H_{i,1}] + \\
+ \frac{1}{(1+R)^3}[P_{h,2}H_i - N_{i,2}],
\end{aligned}
\tag{HHP'}
$$

subject to,

$eq.(1) - eq.(11)$,

$$
\tilde{P}_{n,0}(N_0) \equiv \begin{cases} P_{n,0}(N_0) & \text{if } N_{i,0} > N_{i,-1} \quad \text{and } P_{n,0}(N_0) \geq 1, \\ P_{n,0}(N_0) & \text{if } N_{i,0} < N_{i,-1} \quad \text{and } P_{n,0}(N_0) \leq 1, \\ 0 & \text{otherwise.} \end{cases} \tag{15}
$$

$$
\begin{aligned}
N_{i,1} \equiv \arg\max_{N_{i,1}} \frac{1}{(1+R)^2}[\int_{N_{i,0}}^{N_{i,1}} N_{i,1}\tilde{P}_{n,1}(N_1)dN_{i,1} + \alpha_{r,i,1}H_{i,1}] + \\
+ \frac{1}{(1+R)^3}[P_{h,2}H_i - N_{i,2}],
\end{aligned}
\tag{16}
$$

subject to,

$eq.(1)_{t=1} - eq.(11)_{t=1}$,

$$
\tilde{P}_{n,1}(N_1) \equiv \begin{cases} P_{n,1}(N_1) & \text{if } N_{i,1} > N_{i,0} \quad \text{and } P_{n,1}(N_1) \geq 1, \\ P_{n,1}(N_1) & \text{if } N_{i,1} < N_{i,-1} \quad \text{and } P_{n,1}(N_1) \leq 1, \\ 0 & \text{otherwise.} \end{cases} \tag{17}
$$

In the analytical approach every havven holder knows the demand function for nomins and uses it to choose $N_{i,t}$ taking into account the changes on $P_{n,t}$, $f(P_{n,t})$, $C_{i,t}$ and $C_{opt,t}$ at the same time. We show that in equilibrium, havven holder i maximizes his profits by choosing the number of nomins $N_{i,t}$ that yields $P_{n,t} = 1$. The technical derivation of the result can be found in the Appendix.

The intuition behind the result with respect to profits from fees is the following: a havven holder maximizes his profits from fees when $C_{i,t} = C_{opt,t}$, which occurs when $P_{n,t} = 1$. However, a change in $N_{i,t}$ also affects $C_{-i,t}$. When both $C_{i,t}$ and $C_{-i,t}$ are lower (or higher) than $C_{opt,t}$, a change (in the right direction) in $N_{i,t}$ has a positive effect on $\alpha_{r,i,t}$ through $C_{i,t}$ which is larger than the negative effect through $C_{-i,t}$. When $C_{i,t} < C_{opt,t} < C_{-i,t}$ (or the opposite), a change (in the right direction) in $N_{i,t}$ has a positive effect in both $C_{i,t}$ and $C_{-i,t}$. This last effect is because there is a reduction in $\Gamma_{-i,t}$, increasing $\alpha_{base,t}$.

With respect to profits from seigniorage, the intuition is the following: when there is a need to increase the supply of nomins (i.e., when $P_{n,t} > 1$), a havven holder can improve his profits by selling nomins. The number of nomins he can sell is limited, since no nomin buyer will accept a price greater than $P_{n,t} = 1$. In other words, in this case he maximizes profits from seigniorage when $P_{n,t} = 1$. When there is a need of reducing the supply of nomins (i.e., when $P_{n,t} < 1$), the havven holder has to buy back nomins, which is costly for him. However, future profits from fees will compensate him.

3.2 Calibrated approach:

In this approach, assumptions 4 and 3 do not hold, but assumptions 5 and 6 do. In contrast to the previous approach, we assume that the havven holder does not know the demand function for nomins. This is arguably a step towards a more realistic situation.
Assumption 8. *The inverse demand function for nomins is unknown by havven holders.*

After each additional amount of nomins, the havven holder observes the new market price and issues more nomins until $P_n = \$1$ is reached. He takes into account the prices and the C_{opt} observed in order to choose additional quantities. This is, at period t (which now just indicates "steps" in the calibration process) $N_{i,t} = C_{opt,t-1} * P_{h,t-1} * H_i / P_{n,t-1}$. This number of nomins, together with $N_{i,t}$ will define the new prices $P_{n,t}$, $P_{h,t}$ and $C_{opt,t}$, which are going to be observed again by havven holders to choose $N_{i,t+1}$ and so on. The procedure to change the amount of issued nomins follows the example in the White Paper with the title "Nomins Price Change". This example shows only one iteration. Thus, we recommend to improve the example in the White Paper to clarify this point. It may be possible to just write an explanation or to copy the following calibration (or part of it).

We illustrate the calibrated approach with some examples.

Case 1: Negative shock in demand for nomins.

We suppose there are two havven holders $i = 1, 2$ who possess $H_1 = 100$ and $H_2 = 200$ respectively. Initially, they have issued $N_1 = 50$ and $N_2 = 100$. The interest rate is $R = 0.6\%$ and the fee paid in a transaction with nomins is $k = 0.2\%$. The parameters of function $f(P_n)$ are $\sigma = 95$ and $\phi = 3$, while $C_{max} = 1.25 C_{opt}$ in every period. Havven holders face an inverse demand for nomins given by equation (14) where $\varepsilon GDP = 900$ and $v = 6$, yielding to $P_n = 1$. We also assume that:
Assumption 9. *The velocity of nomins is fixed.*

Notice from equation (14) that when a change in εGDP is completely absorbed by the same change in velocity, the price of nomins is not affected by shocks in the GDP. The assumption considers the extreme case in which a shock has no effect on velocity but on nomins price. This will be discussed further in Section 4.

Important: The parameters were chosen to have $P_n = \$1$. This allows us to start from some steady state and to simulate how the mechanism works after a shock and induces havven holders to recover the stable price. Hence, any change in one of the parameters would imply that the remaining ones have to be adjusted so that $P_n = \$1$ holds. Additionally, the parameters of $f(.)$ were also chosen such that, after a shock, the equilibrium price $P_n = 1$ is reached within 6 iterations or less when both havven holders follow the proposed mechanism. With different parameters, the results may change. This implies that they have to be chosen very carefully for a correct functioning of the incentives.

We start with the initial conditions. Next, we analyze the different strategies for each havven holder after a negative shock in the demand for nomins. These strategies are: (1) Neither havven holder 1 nor 2 adapt their number of nomins, (2) both havven holders adapt their number of nomins, (3) only havven holder 1 adapts N_1, and (4) only havven holder 2 adapts N_2 while havven holder 1 keeps his initial number of nomins.

Initial conditions at period $t = -1$ (before the shock) are:

$P_{n,-1}$	$N_{1,-1}$	$N_{2,-1}$	v_{-1}	$P_{h,-1}$	C_{-1}	$C_{1,-1}$	$C_{2,-1}$	$f(P_{n,-1})$	$C_{opt,-1}$	$C_{max,-1}$
1	50	100	6	1	0.5	0.5	0.5	1	0.5	0.625

Table 1: Initial conditions.

Since, $\alpha_{base,-1} = (v_{-1}kN_{-1})/(H_1 + H_2) = 0.006$ expected profits for each havven holder are:

$$\begin{aligned} \pi_{1,-1} &= \alpha_{base,-1}\tfrac{H_1}{R} = 0.006 \cdot \tfrac{100}{0.6\%} = 100, \\ \pi_{2,-1} &= \alpha_{base,-1}\tfrac{H_2}{R} = 0.006 \cdot \tfrac{200}{0.6\%} = 200. \end{aligned} \tag{18}$$

Finally, note that the amounts of escrowed havvens are $\check{H}_1 = P_{n,-1}N_{1,-1}/(P_{h,-1}C_{max,-1}) = 80$ and $\check{H}_2 = P_{n,-1}N_{2,-1}/(P_{h,-1}C_{max,-1}) = 160$.

1. At the beginning of period $t = 0$, there is a negative shock in εGDP. Since the supply of nomins N has not changed yet and the velocity v is assumed to be fixed, the price P_n is affected (see equation (14)). We consider the case with a drop of 0.9 in εGDP, yielding a new price $P_{n,0} = 0.9$. Thus,

$P_{n,0}$	$N_{1,-1}$	$N_{2,-1}$	v_0	$P_{h,0}$	C_0	$C_{1,0}$	$C_{2,0}$	$f(P_{n,0})$	$C_{opt,0}$	$C_{max,0}$
0.9	50	100	6	0.9	0.5	0.5	0.5	0.905	0.4525	0.5656

Table 2: Negative shock. Neither havven holder changes their number of nomins.

Important: Notice that in Table 2, we obtained a new havven price $P_{h,0} = 0.9$. This, together with the new P_n and old N, implies that Cs do not change. However, since $P_{n,0} \neq 1$, $f(P_{n,0}) < 1$ and $C_{opt,0} < C_i$ there are incentives to reduce N_i. In contrast to our approach, in the example of the White Paper, it is (incorrectly) assumed that $P_{h,0}$ does not change with a demand shock. We recommend to amend the White Paper with corresponding calculations.

If holders decide to not change their number of nomins, $C_{opt,0} < C_{i,0}$ and $\alpha_{base,0} = 0.01$. Hence, holders achieve,

$$\pi_{1,0} = \alpha_{r,i,0}\frac{H_1}{R} = \alpha_{base,0}\frac{C_{max,0} - C_{1,0}}{C_{max,0} - C_{opt,0}}\frac{H_1}{R} = 100, \tag{19}$$

$$\pi_{2,0} = 200. \tag{20}$$

Now, $\check{H}_{1,0} = P_{n,0}N_{1,0}/(P_{h,0}C_{max,0}) = 88.4$ and $\check{H}_{2,0} = P_{n,0}N_{2,0}/(P_{h,0}C_{max,0}) = 176.8$. Thus, the number of "blocked" havvens, which cannot be freely traded without buying back the issued nomins, has increased. In period $t = 1$ (and subsequent periods), if both havven holders continue to keep their number of nomins constant, they make the same profits.

2. Alternatively, they can choose a lower number of $N_{i,1}$ to improve their collection of fees and, as consequence, inducing $P_{n,1}$ closer to one than $P_{n,0}$. So, each holder performs the following calculation to choose his new number of nomins: Because the fee collection is maximized when $C_{opt,1} = C_{i,1}$, they choose $N_{i,1} = C_{opt,1}P_{h,1}H_i/P_{n,0}$ using the price $P_{n,0} = 0.9$, which is the price observed by havven holders. The new nomin price $P_{n,1}$ is given by its demand function for the new $N_{i,1}$ and the velocity. As consequence, the new situation is now

$P_{n,1}$	$N_{1,1}$	$N_{2,1}$	v_1	$P_{h,1}$	C_1	$C_{1,1}$	$C_{2,1}$	$f(P_{n,1})$	$C_{opt,1}$	$C_{max,1}$
0.9945	45.25	90.5	6	0.9	0.500	0.500	0.500	0.999	0.499	0.625

Table 3: Negative shock; both havven holders change their number of nomins following the proposed mechanism.

Notice that $P_{h,1}$ does not change. This can easily be checked using equations (11) and (14). From the former, we know that $vNP_n = \varepsilon GDP$ while form the latter, $P_h = \alpha_c vNP_n/HR$. Since we assume a unique change in εGDP in this exercise, the price of havvens remains fixed after the shock.

Now $\alpha_{base,1} = 0.005$. In this case, holders get,

$$\begin{aligned} \pi_{1,1} &= \alpha_{base,1}\tfrac{H_1}{R} + (N_{1,1} - N_{1,0})P_{n,0} = 86.22, \\ \pi_{2,1} &= 172.45, \end{aligned} \tag{21}$$

$\check{H}_{1,1} = 80$ and $\check{H}_{2,1} = 160$.

The new nomin price $P_{n,1}$ is very close to 1 and, due to the chosen parameters, $f(P_{n,1}) \approx 1$ and $C_{opt,1} \approx C_{i,1}$. Thus, havven holders do not have incentives (i.e., they are collecting the maximum possible fees) to change again the supply of nomins N in order to increase their profits.

3. We consider now the case in which havven holder 1 decides to choose new N_1 (following the mechanism proposed) while holder 2 remains idle.

$P_{n,1}$	$N_{1,1}$	$N_{2,1}$	v_1	$P_{h,1}$	C_1	$C_{1,1}$	$C_{2,1}$	$f(P_{n,1})$	$C_{opt,1}$	$C_{max,1}$
0.929	45.25	100.0	6	0.90	0.500	0.467	0.516	0.966	0.483	0.604

Table 4: Negative shock; only havven holder 1 reacts; $t = 1$

Now, $\alpha_{base,1} = 0.0072$. holders achieve,

$$\begin{aligned}\pi_{1,1} &= \alpha_{base,1}\tfrac{C_{1,1}}{C_{opt,1}}\tfrac{H_1}{R} + (N_{1,1} - N_{1,0})P_{n,0} = 111.79,\\ \pi_{2,1} &= \alpha_{base,1}\tfrac{C_{max,1}-C_{2,1}}{C_{max,1}-C_{opt,1}}\tfrac{H_2}{R} + (N_{2,1} - N_{2,0})P_{n,0} = 174.43.\end{aligned} \tag{22}$$

$P_{n,1}$ is still lower than one, $f(P_{n,1}) \neq 1$, and $C_{1,1} \neq C_{opt,1}$. Thus, havven holder 1 still has incentives to change N_1 (recall we assumed that havven holder 2 is idle). Havven holder 1 will continue to change the number of nomins while he has incentives to do so. Next, we present the results of iterating through subsequent periods:

$P_{n,2}$	$N_{1,2}$	$N_{2,2}$	v_2	$P_{h,2}$	C_2	$C_{1,2}$	$C_{2,2}$	$f(P_{n,2})$	$C_{opt,2}$	$C_{max,2}$
0.919	46.80	100.0	6	0.9	0.500	0.478	0.511	0.951	0.475	0.594

$\Rightarrow$

$\alpha_{base,2}$	$\pi_{1,2}$	$\pi_{2,2}$
0.0074	117.7	173.1

Table 5: Negative shock; only havven holder 1 reacts; $t = 2$

$P_{n,3}$	$N_{1,3}$	$N_{2,3}$	v_3	$P_{h,3}$	C_3	$C_{1,3}$	$C_{2,3}$	$f(P_{n,3})$	$C_{opt,3}$	$C_{max,3}$
0.921	46.52	100.0	6	0.9	0.500	0.476	0.512	0.954	0.477	0.596

$\Rightarrow$

$\alpha_{base,3}$	$\pi_{1,3}$	$\pi_{2,3}$
0.0073	118.2	171.7

Table 6: Negative shock; only havven holder 1 reacts; $t = 3$

...
(See the Excel file for the missing iterations.)
...

$P_{n,6}$	$N_{1,6}$	$N_{2,6}$	v_6	$P_{h,6}$	C_6	$C_{1,6}$	$C_{2,6}$	$f(P_{n,6})$	$C_{opt,6}$	$C_{max,6}$
0.921	46.6	100.0	6	0.9	0.500	0.477	0.512	0.953	0.477	0.596

$\Rightarrow$

$\alpha_{base,6}$	$\pi_{1,6}$	$\pi_{2,6}$
0.0073	118.53	171.58

Table 7: Negative shock; only havven holder 1 reacts; $t = 6$

Havven holder 1 improves his payoffs with respect to the initial stage (see equation (22)) by changing his number of nomins while his rival is idle. His profits improve after each iteration at the expense of havven holder 2's profits. However, P_n stabilizes around \$0.921 instead of \$1. The reason being that, although $P_n \neq 1$, $C_{1,6}$ is similar to $C_{opt,6}$. As a consequence, havven holder 1 is already getting the highest possible amount of fees and has no incentives to change his number of nomins anymore.

4. Finally, we consider the case in which havven holder 1 remains idle and havven holder 2 changes the number of nomins following the proposed mechanism. Again, we present several iterations.

$P_{n,1}$	$N_{1,1}$	$N_{2,1}$	v_1	$P_{h,1}$	C_1	$C_{1,1}$	$C_{2,1}$	$f(P_{n,1})$	$C_{opt,1}$	$C_{max,1}$
0.961	50	90.5	6	0.9	0.500	0.534	0.483	0.994	0.497	0.621

$\Rightarrow$

$\alpha_{base,1}$	$\pi_{1,1}$	$\pi_{2,1}$
0.0064	74.8	197.65

Table 8: Negative shock; only havven holder 2 reacts; $t = 1$

$P_{n,2}$	$N_{1,2}$	$N_{2,2}$	v_2	$P_{h,2}$	C_2	$C_{1,2}$	$C_{2,2}$	$f(P_{n,2})$	$C_{opt,2}$	$C_{max,2}$
0.943	50	93.13	6	0.9	0.500	0.524	0.488	0.983	0.491	0.614

$\Rightarrow$

$\alpha_{base,2}$	$\pi_{1,2}$	$\pi_{2,2}$
0.0063	77.21	203.03

Table 9: Negative shock; only havven holder 2 reacts; $t = 2$

...

(See the Excel file for the missing iterations.)

...

$P_{n,6}$	$N_{1,6}$	$N_{2,6}$	v_6	$P_{h,6}$	C_6	$C_{1,6}$	$C_{2,6}$	$f(P_{n,6})$	$C_{opt,6}$	$C_{max,6}$
0.939	50	93.8	6	0.9	0.500	0.522	0.489	0.978	0.489	0.612

$\Rightarrow$

$\alpha_{base,6}$	$\pi_{1,6}$	$\pi_{2,6}$
0.0063	77.25	204.9

Table 10: Negative shock; only havven holder 2 reacts; $t = 6$

In this case, havven holder 2 improves his profits (Table 10 vs. Table 8) at expense of the other havven holder's profits. Again, P_n does not stabilize at 1 but does at a price closer to 1 than in the previous case, since havven holder 2 has more impact over the supply of nomins.

In summary, both havven holders rather change the number of nomins than remaining idle. Although for each of them the best scenario would be if the rival does not do anything while they adjust their number of nomins, this scenario cannot be an equilibrium. This can be seen from the following strategic game representation of the previous analysis (for this representation, we assume that all iterations are made instantaneously and simultaneously by both havven holders).

	$N_{2,0}$	N_2^*
$N_{1,0}$	100 , 200	77.25 , 204.9
N_1^*	118.53 , 171.58	86.21 , 172.43

Table 11: Negative shock; strategic game representation

$N_{i,0}$ is the action of remaining idle taken by holder i i.e. he chooses to continue having the initial number of nomins in the market. N_i^* is the action of changing the number of nomins following the proposed mechanism. Each box has the payoff that both holders get by choosing some particular action. For example, if havven holder 1 chooses $N_{1,0}$ and holder 2 chooses $N_{2,0}$, the former gets a payoff of 100 and the latter 200. It can be checked that havven holder 1 will choose N_1^* no matter what action is chosen by havven holder 2 (i.e., 1 gets larger payoffs following N_1^* for any action that 2 can take). Similarly, 2 will choose N_2^* no matter what the action of havven holder 1 is. In other words, action N_i^* strictly dominates remaining idle with $N_{i,0}$. Therefore, $\{N_1^*, N_2^*\}$ is the unique Nash equilibrium.

Moreover, this equilibrium yields a stable nomins price $P_n = \$1$ in accordance with the project's claim.

Case 2: Positive shock in demand for nomins.

Since this case is symmetric to the case of a negative shock except for the sign of the shock, we only present the final payoffs of the strategic game. Please see the Excel file for a list of results across all iterations.

	$N_{2,0}$	N_2^*
$N_{1,0}$	100 , 200	100 , 218.5
N_1^*	110.2 , 200	114.7 , 229.5

Table 12: Positive shock; strategic game representation

As in the previous case, action N_i^* strictly dominates remaining idle with $N_{i,0}$ and a unique Nash equilibrium $\{N_1^*, N_2^*\}$ arises. This equilibrium yields a stable nomins price $P_n = \$1$, as can be seen in the Excel file.

Case 3: Positive shock - One of the havven holders plays in the opposite direction of the proposed mechanism.

We consider a case in which havven holder 1 chooses to follow the proposed mechanismin $t = 1$ after observing a positive shock in the demand for nomins, i.e., an increment in nomins price. However, havven holder 2 decides to reduce his number of nomins instead of increasing them and help reducing P_n. He does so with the intention of increasing P_n even more and selling nomins in $t = 2$ at a higher price so that he can gain profits from "seigniorage" at the expense of gains from fees.

Since the initial price $P_n > 1$, holder 2 will need to pay that price in order to reduce his number of nomins in circulation. However, by design, he cannot buy nomins at a price larger than 1. As consequence, he cannot follow this strategy. If he wants to keep the price as high as possible, he has to remain idle, something we have analyzed above.

The scenario with a negative shock in which one of the holders plays in the opposite direction to the mechanism proposed is symmetric to this one. Therefore, there is no room to manipulate the market price in order to achieve gains from "seigniorage".

4 Discussion

The mechanism devised by Havven should be designed in such a way that each player has two kinds of incentives: (i) participation, and (ii) compatibility. The former indicates that the mechanism must give enough incentives to the players to participate instead of doing something else. The latter requires that each participant behaves as intended. In this section, we discuss these incentives (and their related assumptions) for all three players in the following order: the foundation, nomin users and havven holders.

Incentives for the foundation: The foundation is the designer of the mechanism. It has the duty to keep track of all transactions, prices, collateralization ratios, and other parameters, which are crucial for the stability of the system. Additionally, it may propose changes in the set of incentives which govern the system or any other correction needed to improve its performance.

It is assumed that the foundation is a "benevolent dictator" i.e. the foundation does not require any incentives to fulfill its role and it will act in the best interest of all players. Absent this assumption, one would have to investigate incentives for the foundation to take-the-money-and-run under certain circumstances.

Incentives for nomins users: With regard to nomins users, we have assumed that the demand for a stable token exists. This is a strong assumption since nomins are not a "traditional" currency. In particular, there is no national authority backing it or imposing nomins as the official currency. In addition, the equilibrium in the exchange market (the "interest rate parity condition") requires that deposits in different currencies offer the same expected rate of return. Thus, variations in the rates of return for one currency imply fluctuations in its exchange rate to other currencies. The exchange rate between nomins and USD is fixed with $P_n = 1$. Since there is no rate of return from deposits in nomins but there is a positive rate of return from deposits in USD, it is unclear why there would be a positive demand for nomins.

However, users of cryptocurrencies usually look for characteristics, which "traditional" currencies cannot offer. These can be low transaction costs or free and anonymous transnational money transfers. Evidence of this is the demand observed for other stable cryptocurrencies. For instance, Tether had USD 2 billion in circulation in January 2018.

We use the quantity theory of money as the approach to give an explicit expression to the inverse demand function (see equation (14)). It relates the nomin price with some fraction of the total amount of world e-commerce, the number of nomins in circulation, and its velocity. When εGDP increases, it is necessary to increase the supply of nomins and/or its velocity to reduce the price of nomins. Incentives are designed to induce variations in the former while variations in the latter are completely exogenous. How the velocity will react to changes in demand is unknown and depends on the market's distinct characteristics. Note that only in the (unlikely) case in which the change in velocity absorbs all the change in εGDP, it will not be possible to induce price changes with the number of nomins in circulation. Thus, in the "calibrated approach" we assumed a fixed velocity just to focus on the number of nomins. The spreadsheet with simulations allows for a velocity that partially responds to changes in εGDP.

Incentives for havven holders: The participation of havven holders depends on the expected return from fees, which depends again on the fee paid for each nomin transaction and the expected number of transactions. Similar to any investment decision, havven holders only participate if the expected return exceeds the opportunity costs. The fee paid for each nomin transaction is not yet determined, and since there is no estimate for the expected number of transactions, an assessment of whether havven holders will have enough incentives to participate is not possible.

Regarding the compatibility of the incentives, we have shown in Section 3 that, under certain assumptions and parameters, havven holders will follow the proposed mechanism and (in equilibrium) the price of nomins will be close to $P_n = \$1$.

Important: We emphasize the importance of the assumptions and the choice of parameters underlying the obtained results. Specific issues to consider are the following:

- Equation (14) proposes a function in which the price of nomins is strictly decreasing with N. Thus, that function allows for $P_n > 1$ (even $P_n >> 1$) for sufficiently small number of nomins. However, it seems unlikely that a user will accept a large price (e.g., $P_n = 2$) since its last resort buyer (a havven holder who needs to burn nomins) will not pay more than $P_n = 1$ (recall that a havven holder is not allowed to pay more than $1\,USD$ for a nomin). In the extreme case in which $P_n = 1$ for $N \in [0, N^*]$ and $P_n = \varepsilon GDP/(vN)$ for $N > N^*$, the proposed mechanism may fail to incentivize havven holders to increase the number of nomins in circulation. In an extreme case of $N = 0$ the price of nomins will still be fixed at $P_n = 1$. Since, a havven holder collects fees depending on the number of havvens he has (and not in the number of escrowed havvens), he has an incentive to not issue N if he knows that the remaining havven holders have nomins in circulation (and, as a consequence, the system is collecting fees). As was mentioned above, a possible solution might be to make the total fees to be paid to havven holders (equation (8) proportional to the escrowed havvens and not just to the total number of them.
- All parameters must be carefully chosen since, for example, P_h and α_c are connected.
- The function $f(P_n)$ deserves special attention. The value of σ determines how large the range of P_n is such that $f(P_n) = 0$. Therefore, under some negative shocks inducing important drops in P_n, it will be $f(P_n) = 0$ and $C_{opt} = 0$. The effect of this issue in the system is difficult to analyze under numerical examples since $N = 0$ and since it is not possible to calculate P_n from equation (14). Although a large P_n should be expected inducing a large $f(P_n)$, it will remain $C_{opt} = 0$ because $N = 0$ and $C = 0$.
- In the same equation ϕ indicates how "flat" $f(P_n)$ is around $P_n = 1$. The larger ϕ, the flatter $f(P_n)$ becomes. This implies that the incentive to change the number of nomins would be weaker for a larger range of P_n on both sides of 1.
- The (partial) floating design of C_{opt} and C_{max} may produce some problems in very extreme cases. We argue that the design is not fully floating since C_{max} can take a maximum value of 1 and, in this case, C_{opt} a maximum of $1/a$. C_{opt} and C_{max} increase when P_n suffers a positive shock (i.e. price increases). An important positive shock may produce that C_{max} reaches its maximum, implying that C_{opt} will be fixed, and C_i (which also increases with P_n) may be larger than C_{opt} in cases in which the right incentives scheme needs it to be lower than C_{opt}. The system will not suffer this collapse provided that it can react fast to shocks in demand (at least as fast as the shock is produced).

Assumption 4 is necessary for the analytical approach to be tractable. Assumption 5 is necessary to avoid a situation in which incentives go in the wrong direction. Assumption 6 allows havven holders to react to other players. Although this assumption is not made explicitly in the White Paper, we consider that it is reasonable in the sense that it will not be difficult to make N_i "public" from a technical point of view.

Assumption 7 is crucial for the analysis. At some point of the analysis, a specific function for the demand of nomins has to be assumed to determine the price of nomins P_n for a certain supply of nomins.

In the analytical approach we assumed that each havven holder knows the demand function for nomins and anticipates the price of nomins P_n for each number of nomins he issues based on the number of nomins issued by all other havven holders. In the calibrated approach, havven holders do not know the demand function, however, we still assume a specific function. In this case, they decide about the number of nomins they issue based on the observation of previous P_n. After every havven holder has chosen their new number of nomins, a new price P_n is determined and revealed to all havven holders. The havven holders use this information to re-adjust their nomins again and so on. This last approach seems closer to the real world as no player (not even the foundation) knows the demand function for nomins.

Important: The fact that the actual demand function for nomins is unknown generates a critical situation (especially during the implementation stage), since an inadequate selection of parameters may induce an immediate collapse of the whole system. The foundation needs to define a clear and careful road map to reduce this risk. An example of a "dead starting" may be the following: the foundation puts $H = 1000$ on sale at a price $P_h = 0.5$. The parameters of $f(.)$ are $\sigma = 50$ and $\phi = 3$ (such large σ implies that $f(.) = 0$ for sufficiently low $P_n \neq 0$). Suppose that havven holders start to sell nomins under the assumption that there is a strong demand for them. If the assumption is wrong and the demand is weak, there will be an over supply of nomins, inducing a low P_n. Due to the parameters chosen for $f(.)$, this price yields to $f(P_n) = 0$. Hence, $C_{opt} = 0 = C_{max}$. However, since $P_n \neq 0$, we have $C \neq 0$ and we are in a case with $C > C_{max}$.

5 Simulations

In this section we run numerical simulations to stress test the mechanism. We do not consider the behavior of individual havven holders, but rather test how the system as a whole reacts to external demand shocks.[7] This is necessary since the demand function is unknown and might be subject to sudden frictions.

To proceed, we need to start defining the initial inputs. For instance, we consider $C_{opt} = 0.5$, $P_h = 0.5$, $R = 0.5\%$, $\alpha_c = 0.2\%$, etc. Next, some other initial inputs must be adjusted to be coherent with them: the level of adoption ε is chosen such that $P_h = 0.5$, the number of nomins N such that $P_n = 1$, and the velocity v such that $C = C_{opt}$.[8] It is possible to

[7] We have already shown formally that each havven holder has an incentive to follow the proposed mechanism proposed

[8] Since $P_n = \varepsilon GDP/vN$ and $P_h = \alpha_c \varepsilon GDP/HR$, hence, $v = R/(\alpha_c C_{opt})$.

run a simulation with different initial parameters (C_{opt}, fees, initial demand, etc..) but, in such a case, the remaining ones must be re-calibrated.

Furthermore, we make the following assumptions:

1. The inverse demand function is given by equation (14).
2. The adoption rate for nomins (ε) (i.e., the proportion of the total world e-commerce that is done with nomins) is exogenous and hence is exposed to exogenous shocks. We do not consider the possibility that it may be affected by the nomins' history of price volatility. Since we have shown that in equilibrium $P_n = 1$ after every shock, we would not gain anything by endogenizing the adoption rate.
3. We expect the velocity of nomins to be similar to the velocity of USD. However, since we allow the velocity to absorb a fixed part of the change of the level of adoption, it may increase to larger values. We have not considered a more sophisticated velocity function that also depends on the number of nomins in circulation (i.e., that it decreases with a larger supply of nomins and vice versa.)
4. The foundation intends to implement a fixed fee per transaction (α_c) but have not yet decided on the value. Our work relies on the assumption that this value will never change even under an increment in the number of transactions with nomins. Moreover, since the value of α_c is intended to be very low from the beginning, we do not expect any impact on the demand for nomins due to small changes in α_c.

We start by exemplifying a non-stochastic growth and decline in demand. There are ten "years" of growth/decline in demand. Each year starts with a shock after which the system reacts with several steps. At the end of the process, we show that the nomin price approaches the goal of $P_n = 1$. The degree of approximation to $P_n = 1$ depends, among other things, on the parameters of $f(P_n)$ as is shown in the following graphs.

Notice that a larger σ allows for a closer approximation of $P_n = 1$. However, as mentioned above, the risk of $f(P_n) = 0$ increases if there is a sufficiently large negative shock. The degree of approximation also depends on the value of α_c. the lower α_c, the closer the limit P_n to 1.

The non-stochastic decrease shows that after a sufficiently large sequence of negative shocks (in our example a decrease of 10% over 10 "years"), the system collapses. This is not surprising since at "year" 10 only 10% of the demand remains. Since the examples considers a $\sigma = 10$, we get $f(P_{n,10}) = 0$ as was explained before.

Finally, we run a stochastic simulation, which emulates a volatile growth trend for the demand. This case confirms that the stability of the set of incentives depends on the parameters, in particular on σ. The simulation is executed so that the system responds to shocks or jumps in demand from one "year" to the next one. If the system reacts fast enough, it will face more granular variations and it should be capable of absorbing the different shocks. This shows a weakness in the design that could be used by an attacker.

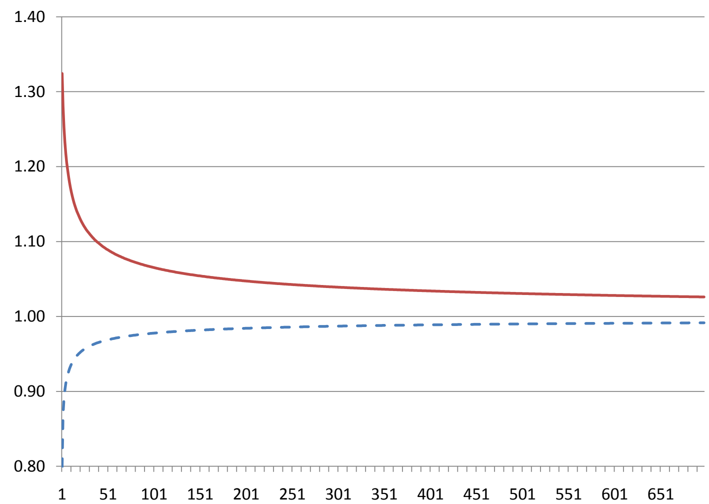

Figure 1: Nomins price evolution in year 1 when $\sigma = 10$ (dash line) and when $\sigma = 1$ (solid line).

Using a $\sigma = 1$ and a very low α_c results in a system that is unstable only when exposed to very large shocks (reducing the risk of an attack) and that, in the limit, P_n converges very close to one.

6 Summary of Recommendations

In this section we reiterate the recommendations that were given in the various sections of the document:

- The calibrated approach in Subsection 3.2 follows the same spirit of the example "Nomins Price Change" from the White Paper, which is made with only one iteration and without assuming a demand function for nomins. This last point implies that it is not possible to know whether the new number of nomins yields to $P_n = 1$. This example should be improved.
- In the same example from the White Paper, it is assumed that P_h does not change after the negative shock. As we show in the calibrated approach, this price does change. See for example, Table 2.
- In Section 3 "Analysis", we assumed a specific demand function for nomins. It is not possible to know the demanded quantity of nomins at $P_n = 1$ without an explicit

demand function. However, the true demand function is not known. Havven must take this issue very seriously, in particular in the implementation stage. Otherwise,

  - The possibility of an initial scenario with $C_{max} < C_i$ emerges, leading to a collapse of the system in the very beginning.
  - In the case that there is no demand for $P_n > 1$ (recall we assumed that it exists demand in such a case), the set of incentives may be too weak to induce havven holders to increase the number of nomins in circulation. Notice that since $f(.)$ is "flat" around $P_n = 1$, a nomin price above 1 but close to it (e.g., $P_n = 1.01$), will not yield a sufficiently large $f(.)$ to induce a change in the number of nomins. If this price results from a very low N, the system will be overcollateralized. In order to mitigate this issue, we recommend to consider tying incentives to the escrowed havvens rather than to all havvens in possession. However, more analysis is required to understand the optimal incentive scheme.

- The system seems incapable of getting out of a situation in which $C_{opt} = 0$. A special protocol may be needed.
- Parameters are crucial for the stability of the system. They have to be chosen carefully to reduce the risk of collapse from shocks in the demand for nomins. We recommend Havven pays special attention to the parameters affecting $f(.)$:
  - The value of ϕ indicates the range around $P_n = 1$ such that $f(.)$ is ("almost") constant. Thus, this parameter is related to the point mentioned above about the inverse demand function for nomins.
  - The value of σ gives a range from zero to some P_n such that $f(.) = 0$. The larger σ, the larger becomes this range and the larger becomes the probability of $C_{opt} = 0$.

Appendix:

Solution of Analytical Approach:

We solve this problem recursively.

In the last period, the havven holder sells his havvens. In order to do this, he has to buy all his nomins. By Assumption 4, he does it at $P_{n,2} = 1$. Thus, in the last period, there is no strategic decision about the number of nomins and he sells the number $N_{i,1}$.

In previous period $t = 1$, given some N_0, we look for the optimal $N^*_{i,1}$.

Before proceeding, notice that the fees to be paid to havven holders depends on the number, vN. When this number changes to $v'N'$ without a change in the supply $N' = N$, then the velocity v' is affected, i.e., $v' \neq v$. After changing supply N' to N'', the velocity will change again such that $v'N' = v''N''$. Therefore, we can treat vN as a constant.[9]

Therefore, after plugging into the objective function (16) the expression of $\alpha_{r,i,1}$ and $\alpha_{base,1}$, taking derivatives with respect to $N_{i,1}$, and after some operations, we get

$$N^*_{i,1}\tilde{P}_{n,1}(N^*_{i,1}, N_{-i,1}) + \alpha_c vNH_iH_{-i}\frac{\Gamma'_{i,1}\Gamma_{-i,1} - \Gamma_{i,1}\Gamma'_{-i,1}}{(H_i\Gamma_{i,1} + H_{-i}\Gamma_{-i,1})^2}, \tag{23}$$

where vN is a constant.

The function $N^*_{i,1}\tilde{P}_{n,1}(N^*_{i,1}, N_{-i,1})$ is increasing in $N_{i,1}$. Thus, the havven holder i finds it optimal to increase $N_{i,1}$. However, when $P_{n,1}(N_1) = 1$, $\tilde{P}_{n,1}(N^*_{i,1}, N_{-i,1}) = 0$ and the holder i does not have incentives to continue increasing his number of nomins.

The sign of the second term, $\alpha_c vNH_iH_{-i}(\Gamma'_{i,1}\Gamma_{-i,1} - \Gamma_{i,1}\Gamma'_{-i,1})/((H_i\Gamma_{i,1} + H_{-i}\Gamma_{-i,1})^2)$ depends only on the comparison of $C_{i,1}$ with respect to $C_{opt,1}$ and it is independent on the comparison of $C_{-i,1}$ with $C_{opt,1}$. We prove this in the following lemma.

Lemma 1. *When havven holder i wants to increase his payoffs from fees, he increases N_i when $C_{i,1} < C_{opt,1}$ and reduces N_i when $C_{i,1} > C_{opt,1}$, independently of the value of $C_{-i,1}$.*

Proof: Lemma 1. We consider now, the following scenarios: (1) $C_{i,1} < C_{opt,1}$ and $C_{-i,1} < C_{opt,1}$, (2) $C_{i,1} < C_{opt,1}$ and $C_{-i,1} > C_{opt,1}$, (3) $C_{i,1} > C_{opt,1}$ and $C_{-i,1} > C_{opt,1}$, and (4) $C_{i,1} > C_{opt,1}$ and $C_{-i,1} < C_{opt,1}$.

1. If $C_{i,1} < C_{opt,1}$,

$$\Gamma_{i,1} = \frac{C_{i,1}}{C_{opt,1}}, \Rightarrow \Gamma'_{i,1} = \frac{C'_{i,1}C_{opt,1} - C_{i,1}C'_{opt,1}}{C^2_{opt,1}},$$

[9] Most realistically, v might change in a lower proportion than N, being P_n the one who is absorbing the remaining part of the change in the supply of nomins. However, for sufficiently small changes in N, we consider that the assumption is acceptable.

where

$$C'_{i,1} = \frac{P_{n,1}N_{-i,1}}{P_{h,1}H_iN_1}, \qquad C'_{opt,1} = f'(P_{n,1})C_1 = f'(P_{n,1})\frac{P_{n,1}N_1}{P_{h,1}H} \leq 0,$$

since $f'(P_{n,1}) = \partial f(.)/\partial P_{n,1} dP_{n,1}/dN_{i,1}$ with $\partial f(.)/\partial P_{n,1} \geq 0$ and $dP_{n,1}/dN_{i,1} \leq 0$.

If additionally $C_{-i,1} < C_{opt,1}$,

$$\mathit{\Gamma}_{-i,1} = \frac{C_{-i,1}}{C_{opt,1}}, \Rightarrow \mathit{\Gamma}'_{-i,1} = \frac{C'_{-i,1}C_{opt,1} - C_{-i,1}C'_{opt,1}}{C^2_{opt,1}},$$

where

$$C'_{-i,1} = -\frac{P_{n,1}N_{-i,1}}{P_{h,1}H_iN_1}.$$

Thus,

$$\begin{aligned}\frac{\mathit{\Gamma}'_{i,1}\mathit{\Gamma}_{-i,1} - \mathit{\Gamma}_{i,1}\mathit{\Gamma}'_{-i,1}}{\mathit{\Gamma}_{i,1}H_i + \mathit{\Gamma}_{-i,1}H_{-i}} &= \frac{C'_{i,1}C_{-i,1} - C'_{-i,1}C_{i,1}}{C^2_{opt,1}}\frac{f(P_{n,1})}{H}, \\ &= \frac{2N_{-i,1}H}{f(P_{n,1})H_iH_{-i}}, \\ &\geq 0.\end{aligned}$$

2. In the case of $C_{-i,1} > C_{opt,1}$ when $C_{i,1} < C_{opt,1}$,

$$\begin{aligned}\mathit{\Gamma}_{-i,1} &= \frac{C_{max,1} - C_{-i,1}}{C_{max,1} - C_{opt,1}}, \\ &= \frac{a}{a-1} - \frac{C_{-i,1}}{(a-1)C_{opt,1}}, \\ \Rightarrow \mathit{\Gamma}'_{-i,1} &= -\frac{1}{a-1}\frac{C'_{-i,1}C_{opt,1} - C_{-i,1}C'_{opt,1}}{C^2_{opt,1}}, \\ &\geq 0,\end{aligned}$$

since,

$$\begin{aligned}& C'_{-i,1}C_{opt,1} - C_{-i,1}C'_{opt,1} = C_{-i,1}\frac{C_1}{N_1}[-f(P_{n,1}) + f(P_{n,1})'P_{n,1}] \geq 0, \\ \Leftrightarrow \quad & f(P_{n,1})'P_{n,1} \geq -f(P_{n,1}), \\ \Leftrightarrow \quad & \sigma\phi(P_{n,1}-1)^{\phi-1}P_{n,1} \geq \sigma(P_{n,1}-1)^{\phi} + 1, \\ \Leftrightarrow \quad & \sigma(P_{n,1}-1)^{\phi}[\phi\frac{P_{n,1}}{P_{n,1}-1} - 1] \geq 1,\end{aligned}$$

which holds for $\sigma \geq 1$ and $\phi \geq 1$ (recall that $\phi \in \mathbb{N} \nmid 2$).

As consequence $\mathit{\Gamma}'_{i,1}\mathit{\Gamma}_{-i,1} - \mathit{\Gamma}_{i,1}\mathit{\Gamma}'_{-i,1} \geq 0$.

3. We consider now $C_{i,1} > C_{opt,1}$ and $C_{-i,1} > C_{opt,1}$,

$$\begin{aligned}
\Gamma_{i,1} &= \frac{C_{max,1} - C_{i,1}}{C_{max,1} - C_{opt,1}}, \\
&= \frac{a}{a-1} - \frac{C_{i,1}}{(a-1)C_{opt,1}}, \\
\Rightarrow \Gamma'_{i,1} &= -\frac{1}{a-1}\frac{C'_{i,1}C_{opt,1} - C_{i,1}C'_{opt,1}}{C^2_{opt,1}}, \\
&\leq 0,
\end{aligned}$$

   since $C'_{opt,t} \leq 0$.

   Therefore, $\Gamma'_{i,1}\Gamma_{-i,1} - \Gamma_{i,1}\Gamma'_{-i,1} \leq 0$ when $\sigma \geq 1$.

4. Finally, $C_{i,1} > C_{opt,1}$ and $C_{-i,1} < C_{opt,1}$.

   Now,

$$\begin{aligned}
\Gamma'_{i,1} &= -\frac{1}{a-1}\frac{C'_{i,1}C_{opt,1} - C_{i,1}C'_{opt,1}}{C^2_{opt,1}} \leq 0, \\
\Gamma'_{-i,1} &= \frac{C'_{-i,1}C_{opt,1} - C_{-i,1}C'_{opt,1}}{C^2_{opt,1}} \geq 0,
\end{aligned}$$

   yielding $\Gamma'_{i,1}\Gamma_{-i,1} - \Gamma_{i,1}\Gamma'_{-i,1} \leq 0$.

□

The previous lemma shows that, when $C_{i,1} < C_{opt,1}$, profits from fees are increasing with $N_{i,1}$ inducing the holder i to choose the largest possible $N_{i,1}$. On the other hand, when $C_{i,1} > C_{opt,1}$, profits from fees are decreasing with $N_{i,1}$ inducing the holder i to choose the lowest possible $N_{i,1}$.

Since $\alpha_c v N H_i H_{-i}(\Gamma'_{i,1}\Gamma_{-i,1} - \Gamma_{i,1}\Gamma'_{-i,1})/((H_i\Gamma_{i,1} + H_{-i}\Gamma_{-i,1})^2)$ does not have derivative at $C_i = C_{opt}$, we cannot apply the standard approach of making the first order condition equal to zero to get the optimal number of nomins.

However, we know that, from equations (23) profits (both from seigniorage and fees) are increasing with $N_{i,1}$ when $N_{i,1} > N_{i,0}$. This case occurs when the nomin price is larger than 1 and there is a need for increasing the supply of nomins. Moreover, both profits (seigniorage and fees) are maximized when $N_{i,1}$ is such that $P_{n,1} = 1$.

When the price P_n is lower than one and there is a need to reduce the supply of nomins, (i.e., there is a need of $N_{i,1} < N_{i,0}$ since $C_{opt,1} < C_1$), fees are decreasing with $N_{i,1}$. In this case, the havven holder i needs to buy and burn nomins, which incurs a cost. Now, reducing $N_{i,1}$ has a negative effect in the first term of equation (23) and a positive effect on its second term. In the case in which equation (23) is negative with $N_{i,1}$, the holder will find optimal to reduce $N_{i,1,}$.

Since we have (implicitly) assumed that a havven holder finds it optimal to invest in havvens to get a return from fees, he also finds it optimal to buy back some nomins to recover these fees since $N_i P_n < N_i$ when he has to buy nomins and $N_i < P_h H_i$ due to the collateralization ratio. Hence, in this case the havven holder's profits (seigniorage and fees) are maximized when $N_{i,1}$ is such that $P_{n,1} = 1$.

In the first period $t = 0$, havven holder do not only take into account the fees and the seigniorage profits of the current period but also the effect of their selection of N_0 in the next period.

Using previous solutions and taking the derivative of (HHP') with respect of $N_{i,0}$, we get

$$\frac{1}{(1+R)}[N^*_{i,0}\tilde{P}_{n,0}(N^*_{i,0}, N_{-i,0}) + \alpha_c \upsilon N H_i H_{-i}\frac{\Gamma'_{i,0}\Gamma_{-i,0} - \Gamma_{i,0}\Gamma'_{-i,0}}{(H_i\Gamma_{i,0} + H_{-i}\Gamma_{-i,0})^2}] - \frac{1}{(1+R)^2}N^*_{i,0}\tilde{P}_{n,1}(N^*_{i,0}, N_{-i,0}), \quad (24)$$

Note that the first two terms (discount with $(1+R)$) are the same expression as the ones analyzed above. We now have the additional expression discounted by $(1+R)^2$, which indicates the effects of current decision $N_{i,0}$ on next period sale or purchase of nomins.

In case we departure from a condition in which the initial supply of nomins is such that $P_{n,0} > 1$ and there is needs of an increasing supply of nomins, the holder cannot find optimal to oversupply them to reduce last term since because the current profit is discounted at a lower rate than the next one.

Alternatively, when the initial supply of nomins is such that $P_{n,0} < 1$, we know from previous analysis that the first two terms give incentives to havven holder to reduce his nomins $N_{i,0}$ such that $P_{n,0} = 1$. This incentive is now strength by the last term because it reduces the negative profits by seigniorage from reducing $N_{i,0}$.